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A-B-Cs of Sun-Synchronous Orbit Mission Design

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The sun-synchronous orbit is one of the most frequently used orbits for earth science missions. Generally within the field of astrodynamics and mission analysis, the sun-synchronous orbit is understood to be a near polar orbit where the nodal precession rate is matched to the earth's mean orbital rate around the sun. This has the effect of maintaining the orbit's geometry with respect to the sun nearly fixed such that the sun lighting along the groundtrack remains approximately the same over the mission's duration. But many engineers still lack the intuitive understanding of how to go about selecting the basic orbit parameters to meet their science mission needs and what exactly are the subtle effects that drive important small-scale geometric variations associated with the sun-synchronous orbit that relate to satellite and science instrument design. The existing literature and various references are tentative and even ambiguous from source to source. This paper is intended to provide a tutorial for practicing engineers, wanting deeper insight into the key characteristics of sun-synchronous orbits. The paper will develop the background and information necessary to explain what a sun-synchronous orbit is and how it works. Handy back of the envelope equations will be provided to enable the mission analyst and system engineer to do quick and simple calculations for the orbit parameters selection and to compute mission parameters important to satellite design without having to resort to sophisticated computer programs.

I. Introduction

The sun-synchronous-orbit (SS-O) is one of the most commonly used forms of earth orbit for space science missions. Historic examples of such missions include NIMBUS, TIROS, COBE, SME, LANDSAT, and others. More recent earth science missions based on the SS-O are Terra, EO-1, and Aqua. And even now there are several future missions either under development or awaiting launch that will also utilize the SS-O within the coming years for their mission: Aura, CloudSat, CALIPSO, Aquarius, and Orbiting Carbon Observatory. The list of past, present, and future earth orbiting sun-synchronous missions is long and impressive. Given the widespread use of the SS-O, it is worthwhile to review the characteristics that make the SS-O so useful and therefore desirable for scientific applications. It is also productive to describe the process of how one goes about selecting the mission parameters defining the SS-O mission design.

The primary reason for the frequent utility of the SS-O is that it readily provides many desirable orbital characteristics which satisfy key mission requirements. Since the orbital inclination is nearly polar (96.5 – 102.5 degrees), the SS-O provides global coverage at all latitudes (with the exception of just a few degrees from the poles). And because the position of the line of nodes remains roughly fixed with respect to the sun's direction, lighting conditions along the sunlit groundtrack remain approximately the same throughout the mission. This property of fixed nodes is also useful to satellite designers in that it results in a nearly constant thermal environment due to sun exposure for the satellite remaining the same over the mission life. Another property often important to a satellite's thermal design is that the SS-O also provides the mission with a "dark-side" to the orbit which always faces away from the sun and which can sometimes be used to solve otherwise complex thermal problems. Still another useful characteristic is that SS-O altitude can be selected over a wide, desirable range (200 – 1680 km) so that they can accommodate a wide range of satellite viewing geometries and conditions. Within the altitude range, another complementary characteristic is that discrete altitudes can be selected to provide SS-Os with groundtracks which repeat after a fixed interval of days. This repeat groundtrack attribute is useful to scientists in ensuring that global coverage is complete and repeatable over a designated sampling period desired by an investigator.

With the utility and desirability of these first-order orbital characteristics, it is easy to understand why earth resource, meteorological, and climate studies missions (among others) are attracted to SS-Os, at least as an initial consideration for their mission design. And as the record shows, many of them do indeed select a

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SS-O to meet their particular needs. So how does one go about defining the best SS-O to meet the needs of a particular scientific investigation? What are the systematic steps that one can go through that enable the mission analyst and system designer to select the specific parameters for a SS-O best suited to the mission's needs? How does one, without excessive computer analyses up front, make a selection of the SS-O parameters to accommodate a set of science requirements?

With this in mind, there are several objectives for this paper: First, it is intended to aid scientists, satellite designers, and even mission designers (who may have forgotten) in understanding what exactly are the unique conditions that qualify an orbit to be called sun-synchronous? What is the specific mechanism within celestial mechanics that enables a SS-O to exist? Second, the paper will define the terms and parameters frequently used to describe a SS-O. The paper is also intended to convey an intuitive "feel" to designer about the orbital geometries for SS-O's with various parameters and a "feel" for the naturally occurring second-order effects that cause variations in the geometry. With this intuition, the paper should enable one to intelligently go about selecting, as a first guess for further studies, a set of SS-O parameters consistent with the mission's needs. And lastly, the paper is intended to provide some handy equations and algorithms that can be used to quickly explore the design space and trade one parameter against another without having to resort to a sophisticated, large computer programs for the analysis. Moreover, some of the algorithms are handy in quickly deriving answers to frequently asked questions that large computer analyses just don't easily answer.

II. The Perturbations due to a Non-Spherical Earth

What exactly qualifies an orbit to be labeled sun-synchronous? Before we can answer this question, we must first review some basic theory and results from celestial mechanics about how an earth orbiting satellite is affected by the first order perturbations due to the earth's oblateness. Almost all textbooks on the subject (for example Refs. 1, 2, 3, and 4) discuss the theory associated with how an orbit plane is perturbed as a result of the earth's equatorial bulge. This bulge creates an out of plane gravitational force on the orbit causing the orbit to gyroscopically precess. The operative equation describing the rate at which the line of nodes moves owing to this bulge is given by:

$$\dot{\Omega} = -\tfrac{3}{2} J_2 \left(\frac{a_e}{p}\right)^2 n * \cos(i) \qquad \text{(Eq. 1)}$$

where $p = a(1-e^2)$ is the orbit parameter (the semi-latus rectum), $n = \sqrt{\mu / a^3}$ is the mean motion, and i is the inclination. μ is of course the earth's gravitational constant and J_2, the zonal harmonic coefficient, with a value for earth equal to 0.001 082 63. Thus, the nodal rate of precession is a function of the three classic orbital elements, namely, the semi-major axis (a), the eccentricity (e), and the inclination (i). (a_e in the equation is the equatorial radius of the earth.) Moreover, we see from the equation that for inclinations < 90° the node regresses, i.e., moves clockwise as seen from the north, and for inclination > 90° the nodal motion is posigrade, i.e., a counter-clockwise precession as seen from the north. With this equation and by the proper selection of the semi-major axis, eccentricity, and inclination, we can cause the orbit plane to precess/regress at different rates along the equator.

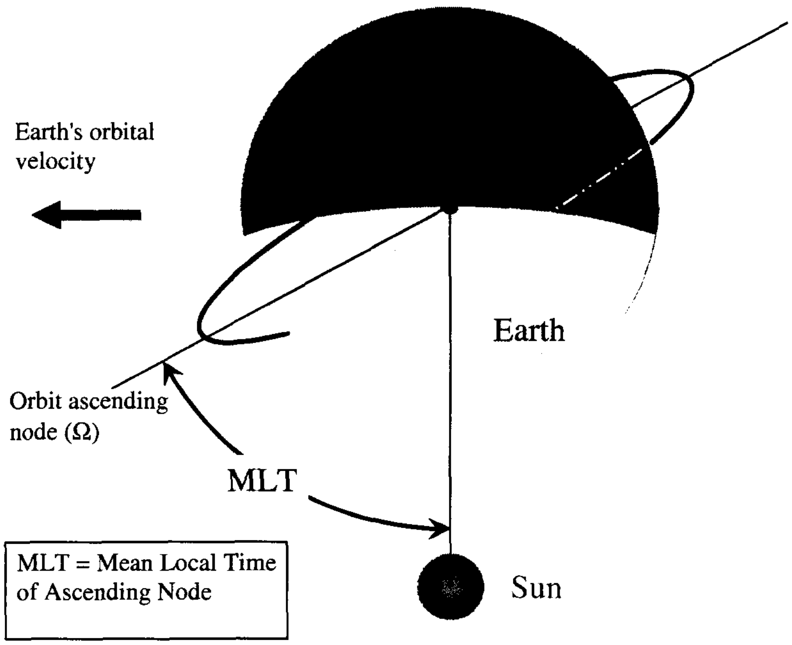
Fortunately, most earth science missions prefer to use circular or near circular orbits (i.e., a "frozen orbit"), thereby pre-determining the value for eccentricity to be zero. (More discussion on the "frozen orbit" will come later.) Therefore, for the analyses in this paper, the orbital eccentricity will be assumed to be zero or approximately zero such that eccentricity may be ignored as a variable in the future applications of Eq. 1. And with this value fixed, the precession rate of the node is reduced to depending only on the inclination and the orbital altitude, $(h = a - a_e)$, through altitude's dependence on semi-major axis.

This relation defines the nodal regression rate of an orbit, any orbit. It tells us that if we can specify values for the inclination and the orbit's altitude, the precession rate, can then be computed. Or conversely, given the precession rate and altitude we easily obtain the inclination. What is needed now is a specification of the nodal rate that gives the property already alluded to as the fundamental characteristic of a SS-O, i.e., maintains the geometry of its nodes fixed with respect to the sun.

III. Selecting the Precession Rate, $\dot{\Omega}$

To simplify this discussion, we start by thinking of an idealization for the earth's motion about the sun. We take the motion to be circular with a period of one year. With this idealization, the rate of revolution would be constant and would given by (360°/365.242199 days =) 0.9856 deg/day. As another approximation, we further consider the earth's polar axis to be perpendicular to the earth's orbit plane (as represented in Figure 1). And because the earth's orbit is posigrade, the motion would appear to be counter-clockwise as seen from the north-pole. For the geometry as shown in Figure 1, it is also to be noted that the plane defined by the earth-sun line and the earth's polar axis, on the sunlit side, define the solar meridian for observers on the earth. (In fact for the approximation at hand, the sun's position would be directly over the equator all year long.) Also, for all points north and south along this meridian, the local solar time would be 12:00 noon. For points west, the local time would be *ante meridiem* (a.m.); similarly for points east, the local time would be *post meridiem* (p.m.). Clearly, 06:00 a.m. would correspond to being on the terminator reckoned to the west of the solar meridian and vice-versa for 06:00 p.m. With these idealizations, we have established a coordinate system and geometry that enable us to describe and define other orbit parameters.

Figure 1: Earth-Sun Geometry Schematic

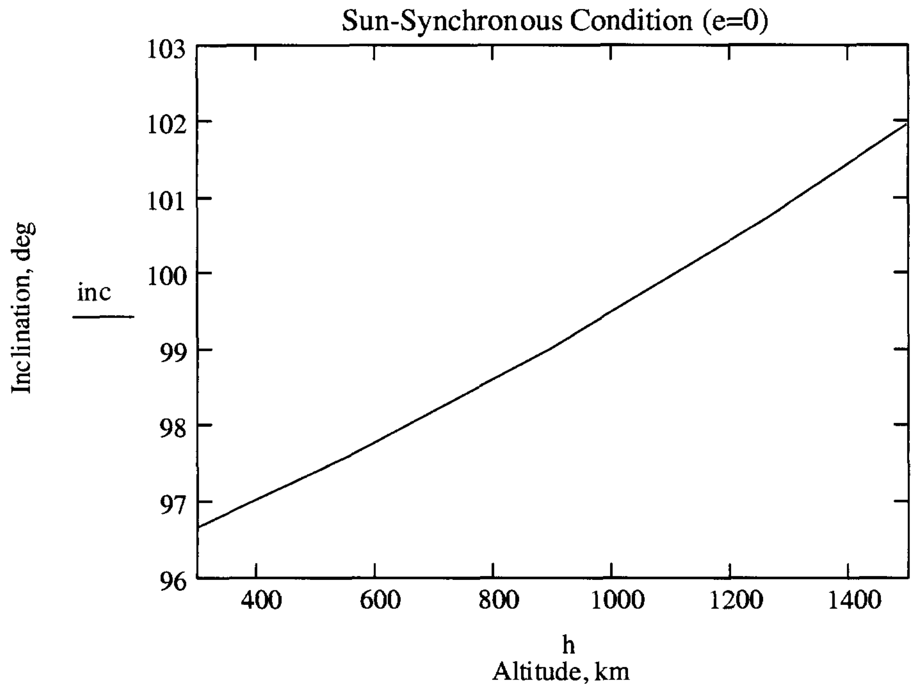


If we now consider a satellite with its ascending node positioned as shown in the figure, we can label the angle between this node and the direction to the sun (Ω). We also call this angle the Mean Local Time of the ascending node (MLT), because the usual way of describing this angle is in terms of time relative to the noon meridian. For an arbitrary orbit, the MLT would continually change owing to the regression/precession phenomenon described above. If, however, we were able to precess the line of nodes at exactly the earth's mean orbital rate around the sun, the geometry shown in the figure would be preserved and the MLT angle with respect to the sun-line would remain constant. Thus, the selection of earth's mean orbital rate for $\dot{\Omega}$ would, therefore, achieve the desired result of maintaining the orbit's geometry with respect to the sun fixed and the sun-lighting conditions along the orbit's groundtrack the same. In fact as shown in Figure 1, we would have a SS-O with its ascending node at ≈08:00 a.m. MLT (or equivalently -60 degrees).

Hence, the condition for an orbit to be sun-synchronous is that the precession rate equals the earth's Mean Motion. When this is done, the specification of the orbit's altitude yields the inclination automatically as defined by Eq. 1. Figure 2 below shows how the orbital altitude and inclination for SS-Os are uniquely paired, i.e., selecting one determines the other, over an altitude range of general interest to earth scientists.

Thus a scientist or system engineer wanting to define orbital parameters for a SS-O has available the orbit's altitude and the orbit's MLT angle. (Here again we are still assuming that only circular orbits are of interest.) From the orbit altitude, the inclination is automatically specified through Eq. 1 as plotted in the curve in Figure 2. Specification of the MLT parameter defines the sun lighting conditions for points beneath the orbit as well as the duration of time spent in eclipse by the satellite. More on this later.

Figure 2: Sun-Synchronous Condition: Inclination vs. Altitude (e=0)



IV. Reckoning Time

Any textbook on astronomy, most notably a book on spherical astronomy (e.g., Refs. 5 and 6), quickly makes clear that the reckoning of time is a complex subject. Yet understanding Mean Solar Time and how it is reckoned is critically important to understanding the finer points of SS-O mission design.

The time interval between meridian transits of the real sun for an observer defines the apparent solar day. However, the use of the apparent solar day as a basis for timekeeping is inconvenient. This is because the sun's eastward movement against the background stars varies from day to day, making the length of the apparent solar day as reckoned by solar meridian passages variable. This variability is due to the earth's elliptic orbit and the obliquity of the ecliptic. Fortunately these variations average out over the course of a year to be very nearly zero and this average value defines the length of the mean solar day at 86400 seconds.

In order to create a concept for the mean solar day similar to the apparent solar day, the mean solar day can be thought of as the time interval between two successive transits across an observer's meridian of an imaginary body referred to as the "mean sun". This mean sun is a fictitious body that moves along the celestial equator at a rate of advance equal to the earth's Mean Motion. The mean sun completes one circuit around the earth in identically the same time interval as it takes the real sun to transit its path from vernal equinox to vernal equinox along the ecliptic. This would be the Tropical year (365.242199 days). Thus, the mean sun is not unlike the idealization of the sun's motion relative to the earth described earlier in Figure 1. But more importantly, the mean solar day is of fixed length corresponding to the interval of exactly one mean solar transit across the noon meridian to the next. It is for this reason that civilian populations use Mean Solar Time as the basis for reckoning time. Mean Solar Time and its fundamental unit, the mean solar day, are in effect the time standard against which all other phenomena are measured.

Now on any given day, the real sun will cross an observer's meridian at a time different from the noon hour as measured by a clock. This difference between apparent solar time and Mean Solar Time is called the Equation of Time. And since the parameters for the earth's orbit are known quite well and since the obliquity of the ecliptic is also known well, it is possible to compute the Equation of Time as a function of calendar day. Moreover, because the orbital parameters for the earth's orbit change slowly and the precessional motion of the vernal equinox is also quite slow, the Equation of Time for a given date in the year would remain the same from year to year were it not for leap years. It is for this reason that algorithms to compute the Equation of Time require a calendar date as an input. As we will see, this is only a minor inconvenience for the applications detailed later in the paper.

Before leaving the subject of "time", it is useful to talk about another parameter important to SS-O mission design. In particular we are interested in defining the rate at which the earth rotates with respect to the vernal equinox. We know that at some particular instant in time the mean sun and the vernal equinox are coincident. After one Tropical Year they are coincident again. Over this interval, the earth has rotated 365.242199 revs with respect to the mean sun, but one more time with respect to the vernal equinox. As a result, we see that the period of time, τ, in which it takes the earth to rotate once with respect to the vernal equinox is given by:

$$\tau = 86400 * \left(\frac{365.242199}{1+365.242199}\right) = 86164.09 \text{ seconds}$$

Therefore, the earth's rotation rate with respect to the vernal equinox is given by

$$\omega_e = \frac{360^o}{\tau} = \frac{360^o}{86400}\left(1+\frac{1}{s}\right) \qquad \text{where } s = 365.242199$$

Now our ultimate goal here is to develop a relationship for the rate at which earth longitude advances for a SS-O as it moves around its orbit. This is important to computing groundtracks for SS-Os. We know that if the earth did not rotate the change in longitude between one nodal crossing and the next would be:

$$\Delta L = \omega_e * P \qquad \text{where } P \text{ is the nodal period.}$$

However the orbit plane precesses along the equator at the rate described in the previous section. So when we include orbital precession in evaluating the change in longitude in one period, we have:

$$\Delta L = \left(\omega_e - \dot{\Omega}\right) * P = \frac{360^o}{86400} * \left(1+\frac{1}{s}-\frac{1}{s}\right) * P = \frac{360^o}{86400} * P$$

Thus, for a SS-O, the rate of longitude advance as measured along the earth's equator is identically 360 degrees in one mean solar day. This is a handy and useful relationship unique to SS-Os when evaluating how fast the groundtrack advances in longitude as a function of time.

V. System Engineering the Mission Design

Consistent with good system engineering practice, the first step in formulating a mission design (whether for a SS-O mission or otherwise) is to gather science and mission requirements. Clearly, the first point of contact would be with the mission's science team to glean an understanding of the science objectives and science requirements from which other mission requirements would be derived. A second step would be to contact the mission's sponsor or program office to collect any high-level guidance, constraints, and/or directives to be imposed on the mission design. These too could translate directly into mission requirements. Next, it is always advisable to inquire about basic system considerations such as launch vehicle capability and launch constraints, inherent satellite and/or instrument capabilities, tracking facilities and ground stations to be used in data recovery, and any other inherited system capabilities that could map back into requirements on the mission design. Following this, these requirements, constraints, and capabilities would all be analyzed to derive a succinct, but traceable, set of requirements to be the basis for selecting orbit parameters characterizing the mission design.

Science requirements, probably more than any other inputs, tend to dictate the mission requirements that drive the selection of specific orbit parameters. Statements by the scientist about the mission lifetime, the need for a certain amount of geographic coverage, the frequency with which the coverage is to be repeated, the need to access specific targets, the acceptable distances over which measurements of targets are to be made, and even seasonal preferences for certain observations are the kinds of statements desired and the kinds of statement that most readily map into a specific mission design. (See Table 1 below.) And after collecting these statements, it then becomes important to carefully analyze how they flow-down into one or more mission requirements. One science requirement/desire can relate to the selection of several orbit parameters, e.g., altitude, inclination, geographic location of an ascending node. These broad statement need to be traced with a connectivity to as many specific mission requirements as necessary to ensure the mission designer knows all that is expected. Moreover, the orbit design process is rarely a one-pass operation, but rather it is usually iterative where the choice of orbit parameters is generally determined only after extensive analyses and trades to further understand how the selected parameter best suit the mission's needs and only compromises the satisfaction of science objectives to an acceptable degree. These analyses will also enable an understanding of the sensitivities and resiliency of the mission design to possible future variations in the selected parameters.

Frequently scientists are reluctant to be specific and/or quantitative about what they really want. It then becomes the system engineer's responsibility to draw-out these specifics by first developing an understanding of what is desired and then by describing the conditions provided by a particular solution for the orbit and its ability to achieve science objectives. This is frequently that core of the iterative cycle alluded to above.

Table 1: Science Requirements/Desires Mapped to Orbit Characteristics

Stated Science Requirements/ Desires	**Motivating Objective or Instrument Characteristic**	**Traceable Orbit Characteristic**
Limitation on the range to a target; viewing angle constraints;	Instrument sensitivity, resolution, field of view/swath-width, allowable elongation/ distortion over a footprint, etc.	Orbit altitude
Number and distribution of targets to be observed (for discrete targets)	Unique geographic targets to be measured	Orbit altitude, inclination; groundtrack grid density; groundtrack tied point to achieve over-flight of specific lat/lon
Area coverage to be provided (for continuous targets)	Percentage of earth's surface to be accessible for observation	Orbit inclination, altitude
Frequency with which targets/areas are to be sampled	Allowable time interval before a repeat observation is possible	Orbit altitude
Sun-lighting conditions to be provided (for optical measurements)	Consistent sun shadows for targets	Orbit nodal position and/or nodal Mean Local Time; orbit inclination
Seasonal considerations of observations	Visual access to Antarctica (for example) during Antarctic summer	Orbit nodal position and/or nodal Mean Local Time; orbit inclination
Overall duration/period of time necessary to measure some phenomenon through its life-cycle	Life expectancy for instrument, system; mission life, operations duration, total volume of data, etc.	Orbit altitude

VI. The Use of "Frozen Orbits"

Once a set of mission requirements has been derived and analyzed and unless it is very clear that the mission demands an elliptic orbit with a significant eccentricity, the most straightforward way of initiating the selection of the orbit parameters is to assume *a priori* a circular orbit. In practical applications, truly "circular" orbits around the earth don't really exist. Rather mission designers opt to use the "frozen orbit" as the closest realizable approximation to the theoretical circular orbit. The concept of a frozen orbit is derived from the clever use of perturbation theory and judicious choices for the eccentricity, e, and argument of perigee, ω.

It is well known from perturbation theory [Refs. 7 and 8] that both e and ω generally vary as a function of time. However, by the careful selection of the eccentricity and argument of perigee, the equations show that e and ω will vary in a coupled oscillation. This functional dependency yields coupled values for e and ω which evolve as a function of time by moving counter-clockwise around a closed contour in e-ω space over one apsidal period. Thus, by balancing the effects of the J_2 and J_3 perturbations on the orbit, the values of e and ω can be maintained relatively fixed, or "frozen", with generally small oscillations with respect to the selected stable point. Since the value for eccentricity that enables this oscillation is small, i.e., on the order of 0.001 or less depending on the other orbit parameters, the frozen orbit closely approximates the characteristics of a real circular orbit. The nominal values of ω that makes this work are 90 or 270 degrees, thus placing the perigee nearly over (depending on the oscillation) the north or south pole, respectively. For most NASA-sponsored SS-O missions, the argument of perigee is usually selected to be 90 degrees, i.e., orbit perigee over the north pole.

VII. Selecting Orbit Altitude

Accepting the frozen orbit as a basis for specifying the eccentricity, we turn now to selecting the parameter that probably has the greatest traceability to the suite of science requirements: altitude. Orbit altitude unquestionably relates to more mission requirements than do the others. Selecting the altitude or narrowing the range of possible altitudes acceptable to science is a good step.

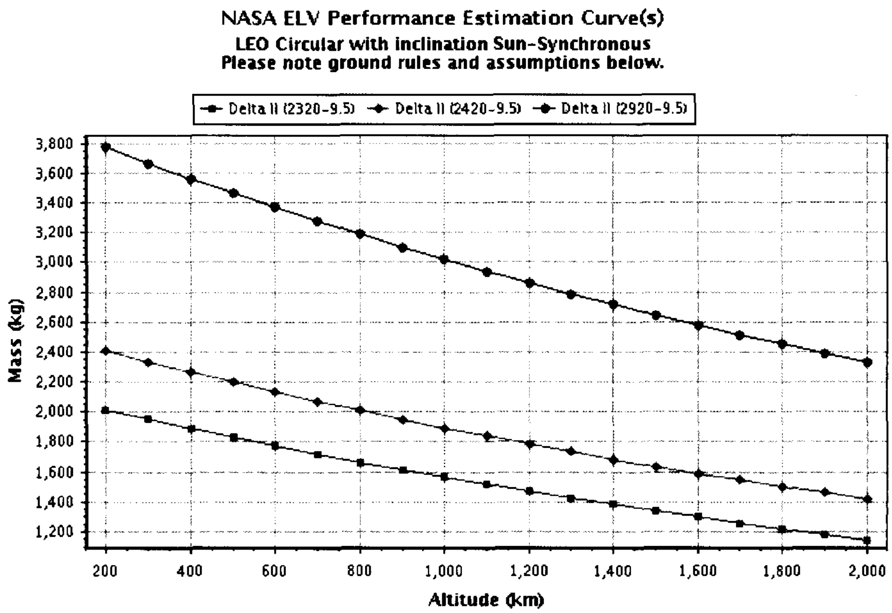
What range of orbital altitudes should be considered for science missions in low earth orbit? Wertz in Ref. 9 defines low-earth orbit as orbits with altitudes below 1000 km. His underlying rationale is that, for orbits with altitudes greater than 1000 km, the Van Allen radiation belts come into play by exposing a satellite to greater and greater amounts of trapped radiation. This, in turn, forces the satellite designer to provide radiation-resistant components and piece-parts tolerant to the expected exposure. Thus, a satellites designed to operate above 1000 km is likely to be life-limited owing to the radiation environment or it will be too expensive to build. Following this reasoning, an upper bound near or just somewhat greater than 1000 km is taken as a reasonable value for fixing the upper bound for low earth orbit altitudes. In this paper, we will consider altitudes up to 1680 km for reasons driven more by coverage and access to ground targets than a strict consideration of radiation.

At the low end of the altitude range there are other considerations in defining an acceptable orbital altitude. This is due to the fact that satellites in very low orbits (200 – 500 km depending on the satellite's ballistic characteristics) can be seriously affected by atmospheric drag. This drag force acts to continuously erode energy from the orbit and slowly decreases the semi-major axis, hence altitude. To compensate for this loss of energy, the satellite must make propulsive maneuvers to re-boost altitude and to restore orbital energy. Thus, one significant consideration when planning to use a low earth orbit is drag compensation which requires the expenditure of satellite propellant as a consumable. Additionally, drag compensation further increases the complexity of the mission operations by requiring the orbital altitude to be constantly monitored. Then, when conditions require, propulsive maneuvers must be planned to raise the orbit, but not without some adverse impact on the science data collection time and on the overall complexity of the mission operations. Therefore, the use of very low altitude orbits, i.e., below 500 km, also has its drawbacks, which tend to make mission designers choose orbit with higher altitudes.

As a practical matter in the selection of orbit altitude, drag affects all SS-O satellites over the range of interest, but at altitudes higher than 500 km, it becomes a much more manageable problem, with small propulsive maneuvers and with maneuvers being relatively infrequent.

Another factor in selecting a mission's altitude is associated with launch vehicle performance. As the curves in Figure 3 show, the mass delivered by each of three versions of the Delta II vehicle decreases as the target altitude becomes increasingly greater. It should be noted that these curves have implicit in them, according to Eq. 1, the variation in orbital inclination as the altitude varies. From the launch vehicle point of view the altitude range defining low earth orbits lies between 200 and 2000 km. Also, if we were to consider other launch vehicles, e.g., Atlas going to a SS-O, a similar functional dependence of delivery mass on altitude would be observed. Thus, for a mission and system designer, specifying the altitude for a chosen SS-O maps directly into an upper bound on the mass allowable for the satellite, and vice-versa. As the figure shows, it is possible to jump from one version of the Delta II to another in order to increase the mass performance, but this jump does not come without incurring greater launch costs.

Figure 3: NASA ELV Performance Estimation Curves



Now we come to what is likely the most important consideration is selecting orbital altitude for a scientific mission using a SS-O. This critical consideration has to do with the viewing "access" provided by the orbit. The term "access" can refer to one of several things related to coverage and viewing geometry. First, it can refer to the actual distance that a satellite must pass relative to or over-fly a given ground target. For example, the scientists might have expressed a requirement that limits the line of sight distance from the instrument to targets to be less than some specified value. This requirement on the measurement distance could also be related to instrument sensitivity and/or field of view (FOV).

In a different context, "access" could refer to the time interval between successive passes that a satellite makes relative to a target. In this case, the scientists may have specified another requirement that expresses the time interval and sampling criteria with which they want to repeat their measurements. This might be important to obtaining data for phenomena which have a variability driven by a time-constant. Another aspect of "access" could refer to the distance between adjacent groundtracks, so as to permit observations of a given geographic location from different vantage points and/or viewing angles, but within some acceptable line of sight distance.

An approach to understanding how we can use access requirements to enable a choice of orbital altitude for a SS-O is to first begin by considering a simple calculation made with Kepler's Equation to bracket the range of altitudes based on the repeatability of an orbit's groundtrack in just one-day. A quick survey shows that there are only five discrete solutions of practical interest to this problem. These five orbits have their orbital altitudes contained in the range between 250 and 1680 km as indicated in Table 2 below. Clearly this range overlaps and is consistent with the altitude range previously discussed in this section as being reasonable for low earth orbit science missions. These five solutions correspond to orbits which make exactly 12, 13, 14, 15, and 16 revolutions in one-day. The unique property possessed by each is that in one-day's time they complete an integer number of revolutions and then begins to repeat their groundtrack over again. This means that each orbit lays down a groundtrack grid on the surface of the earth with 12, 13, 14, etc. ascending nodal positions equally spaced around the equator.

Table 2: Orbit parameters for SS-O with an integer number of revs in one-day

Revs per Day, #	Orbital Period, seconds	Equatorial Altitude, km	Distance between Adjacent GTs, km
12	7200.00	1680.86	3339.59
13	6646.15	1262.09	3082.69
14	6171.43	893.79	2862.50
15	5760.00	566.89	2671.67
16	5400.00	274.42	2504.69

To make this point, let us consider the case of 14 revs in one-day. The nodal orbital period, P, for this example is computed as:

$$P = 86400 / 14 = 6171.43 \text{ seconds.}$$

Solving Kepler's Equation for the semi-major axis and then subtracting the equatorial radius, then gives the altitude:

$$h = (a_e - a) \,, \qquad \text{where } a = \sqrt[3]{\mu\left(\frac{P}{2\pi}\right)^2}$$

The altitude computed for the 14 revs repeat is 893.79 km as shown in Table 2. And now another important piece of information is the distance between adjacent groundtracks, which is computed by dividing the circumference of the earth by 14:

$$2\pi * a_e / 14 = 2862.50 \text{ km} \qquad (a_e = 6378.14 \text{ km})$$

Hence, for this example, it is clear that if the orbit's groundtrack could be positioned to over-fly targets of interest that were separated by 2862 km along the equator, then this altitude would be ideal in providing opportunities for scientific observations every day. Even so, the groundtrack grid provided by this orbit is quite coarse.

In most real applications, the targets are more unevenly distributed over the earth, thus requiring a groundtrack grid of finer mesh. In order to obtain this kind of a grid, we next consider the use of orbits which repeat in exactly two-days instead of one. Starting with the solutions identified for the one-day repeat ranging between 12 and 16 revs, we can easily see that for a two-day repeat there should be solutions corresponding to the integers between 24 and 32 or:

24, 25, 26, 27, 28, 29, 30, 31, 32

At first there appears to be a total of nine possible solutions. But this is misleading, because a simple analysis, for example, for the case of the 28 revs in two-days:

$$P = 2*86400 / 28 = 1*86400 / 14 = 6171.43 \text{ seconds}$$

quickly shows that this orbit has identically the same period as the 14 revs in one-day (see Table 2.). In other words, this is a degenerate case with the one-day, 14-rev solution. (As a matter of convenience, we adopt the notation of 1D14R for the one-day repeat in 14 revs, which by the previous analysis is identical to 2D28R or 4D56R and so forth.)

Now if we consider a non-degenerate case from the list of nine, say 29 revs in two-days (2D29R), we can compute the orbital period as before:

$$P = 2*86400 / 29 = 5958.62 \text{ seconds}$$

Again using Kepler's Equation to compute the semi-major axis, and subtracting the earth's equatorial radius to obtain the orbital altitude, one gets:

$$a = 7103.78 \text{ km or} \qquad h = 725.64 \text{ km}$$

Therefore for this case, the orbital altitude is defined, and from the altitude the inclination is inferred.

The next thing to note about the 2D29R solution is that, since it makes 29 revs in two-days, the groundtrack grid now has 29 ascending nodes around the equator, thus reducing the distance between adjacent ascending nodes to just 1382 km. This means that an imaging instrument with a FOV oriented perpendicular to the groundtrack and centered on the groundtrack could with a swath-width of just 63 degrees provide complete coverage of the entire earth, i.e., global coverage with "access" to every place on earth for imaging, in just two days. We have effectively achieved the sought after finer groundtrack mesh, but at the expense of increasing the "access" interval. With the 2D29R solution, a particular target (with a particular viewing geometry) will be visited once in two-days. (As an aside, for some targets there will be two opportunities for observations of a particular target corresponding to ascending and descending passes relative to that target, but the viewing geometry will be different, i.e., the target will be on the right side of the groundtrack for one pass and on the left for the other.)

If we now extend this process to consider orbits which repeat in 3, 4, 5, 6, 7... days and so forth, then the first result is to realize that the number of possible solutions increases proportionately. For example, for the case of a seven-day repeat groundtrack, we have as possible solutions for the number of orbital revolutions in that interval given by:

$$12*7 = 84, 85, 86, 87, \ldots 108, 109, 110, 111, \text{and } 112 = 16*7$$

for a total of 29. But as before, we must carefully cull out those solutions which are redundant by having the same orbital period as others. When this is done, there remains a total of 24 unique solutions with an integer number of revs in exactly seven-days. It may be of interest to note that had we picked the eight-day repeat cycle, the apparent number of possible solutions would have increased to 33, but the number of surviving, non-degenerate solutions would not have been so plentiful after culling out the non-redundant solutions. This is because 7 is a prime number and 8 is not. Thus, with the eight-day repeat, the number of degenerate solutions is 17 out of 33, leaving only 16 viable. The message here to the mission designer is that the abundance of viable solutions for an integer number of revs in a given repeat cycle is much greater for repeat days being a prime number, i.e., 3, 5, 7, 11, 13, 17, etc. days.

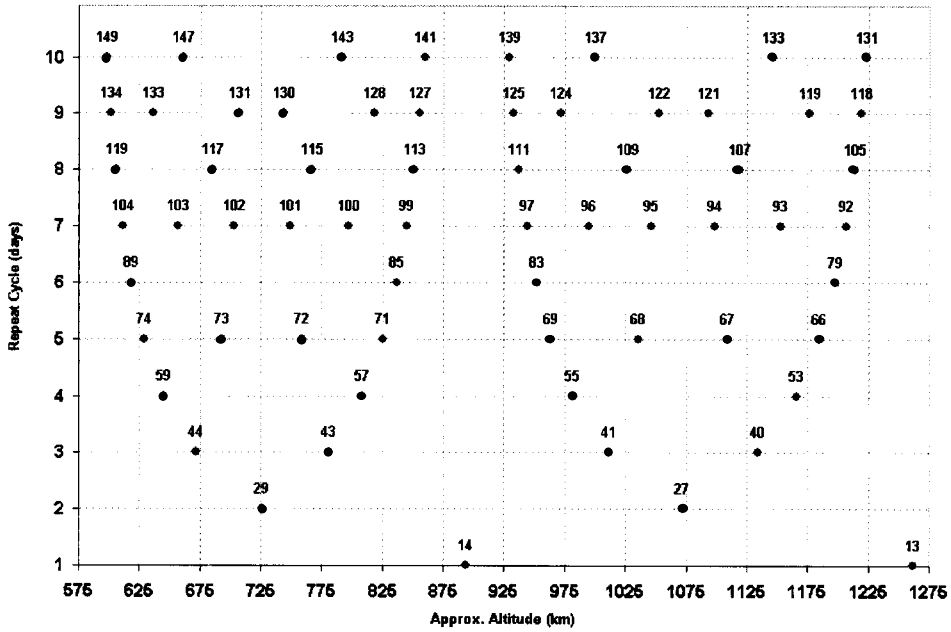
With this knowledge, it is possible to construct a matrix of viable, discrete solutions for according to the days in the repeat cycle versus the number of revs. And as a further step, it is possible to display this matrix in a graphic format as a function of orbital altitude. This is shown in Figure 4, where a subset of the total matrix of data for Repeat Cycles is plotted against orbital altitude (Ref. 10). (It should be noted here that the altitudes shown are approximate, since they are based on Kepler's Equation and do not include the effects of the J_2 perturbations. For preliminary mission design these are more than adequate.)

From Figure 4, we immediately recognize two of the five solutions for the one-day repeat at: 1D13R and 1D14R. Jumping up to the two-day repeat line, we see the solutions: 2D27R and 2D29R. (These solutions are the only two contained in this limited subset of the matrix captured in the figure.) Moving on to the three-day repeat, we see, as expected, that the number of possible solutions increases: 3D40R, 3D41R, 3D43, and 3D44R. And jumping further still to the eight-day repeat, we see solutions: 8D119R, 8D117R, 8D115R, and so forth.

We can see from Figure 4 that an arbitrary altitude for a SS-O will most likely not fall onto one of the discrete solutions plotted in the figure. At the same time, it should be realized that due to the density of rational number along the real number line, an arbitrary altitude will fall onto a rational number corresponding to some number of revs in some number of days to repeat the cycle. The integers here may be quite large, but integers can nonetheless be found. Take for example the altitude of 750 km. From this we compute the period to be 5989.29 seconds, corresponding to 14.42575 revs per day. Diligently searching for a rational number that most closely approximates 0.42575 we find that 453/1064 equals this decimal number to the accuracy expressed. Thus, by a little manipulation we find that R = 15,349 revs in 1064 days. In other words, the SS-O corresponding to 1064D15349R has an equatorial altitude of 750 km. This demonstrates that at some level of accuracy, all orbits, regardless of their altitude, repeat their groundtrack if you wait long enough.

A last interesting point to note here relates to the time interval between the laying down of adjacent groundtracks, since the time between observations is an aspect of orbital "access". Understanding this time interval could be important to satisfying a science requirement related to some minimum time interval between observations of the same target, but from two different vantage points. We ask the question: what is the time interval between the laying down of two adjacent groundtracks?

Figure 4: Sun-Synchronous Repeat Groundtrack Orbits



It should be obvious that adjacent groundtracks are not laid down consecutively. The change in longitude between consecutive nodal crossings for an orbit is given by the so-called "fundamental interval" and was discussed in a previous section as:

$$\Delta L = \left(\omega_e - \dot{\Omega}\right) * P$$

where ω_e is the earth's rotation rate and $\dot{\Omega}$ is the orbit's rate of precession. For the range of altitudes under consideration in this paper, ΔL ranges between 22.5 and 30 degrees. And with a little manipulation, we have ΔL in terms of D and R given by:

$$\Delta L = 360^o * D / R$$

But the difference in longitude between two adjacent nodes is given by:

$$\Delta l = 360^o / R$$

Taking the case for 8D117R as a specific example, we see that there are exactly:

$$117 \text{ revs} / 8 \text{ days} = 14 \frac{5}{8} \text{ revs per day}$$

In other words, it takes approximately 14 revs before the earth has rotated sufficiently to allow the satellite to lay down a groundtrack near the initial ascending node. But because there are an integer number of revs (117) before arriving back at the original ascending node, the misses on consecutive one-day intervals must come in steps corresponding to rational numbers based on the number of days in the repeat, in particular *n* * 1/8 of ΔL where *n* is an integer between 1 and 8. Therefore, after a little more than one day, corresponding to exactly 15 revs, it is easy to show that the longitude difference at the node crossing from the initial node is 9.2308 degrees or exactly 3/8 of ΔL from the initial node. After two days or 29 revs, the miss is –6.1538 degrees, or –2/8 of ΔL. Now after three days, we have (3*(14 and 5/8 revs)= 42 and 15/8

= 43 and 7/8) starting on rev 44 a miss of 3.0769 degrees, or 1/8 of ΔL. Since this miss distance is equal to the value for Δl for this SS-O, we are laying down the adjacent groundtrack to and just east of the first groundtrack. Therefore, for the 8D117R SS-O, it takes 44 revs or 3 days, 12 minutes, and 18.46 seconds until the satellite crosses the equator on a groundtrack adjacent to the first groundtrack. Having found one solution, the time interval for the groundtrack on the opposite side of the original is obtained as:

117 – 44 = 73 revs or 4 days, 23 hours, 47 minutes, and 41.54 seconds.

By following this recipe, one can for an arbitrary case of R-revs in D-days find the time interval between adjacent groundtracks.

In concluding this section, it is worthwhile to list (Table 3.) some handy equations useful in computing orbit parameters for SS-Os as a function of the number of days, *D*, in a repeat cycle and the number of revs, *R*, in the cycle.

Table 3: Handy Equations

Nodal Period, *P*, seconds	P=86400 * D / R
Semi-Major Axis, *a*, km	$a = \sqrt[3]{\mu(P/2\pi)^2}$
Equatorial Altitude, *h*, km	$h = a - a_e$
Fundamental Interval, ΔL, deg	$\Delta L = 360^o * D / R$
Fundamental Interval, ΔL, km	$\Delta L = 2\pi * a_e * D / R$
Interval between Adjacent GTs, Δl, deg	$\Delta l = 360^o / R$
Interval between Adjacent GTs, Δl, km	$\Delta l = 2\pi * a_e / R$

VIII. Selecting the Nodal Position

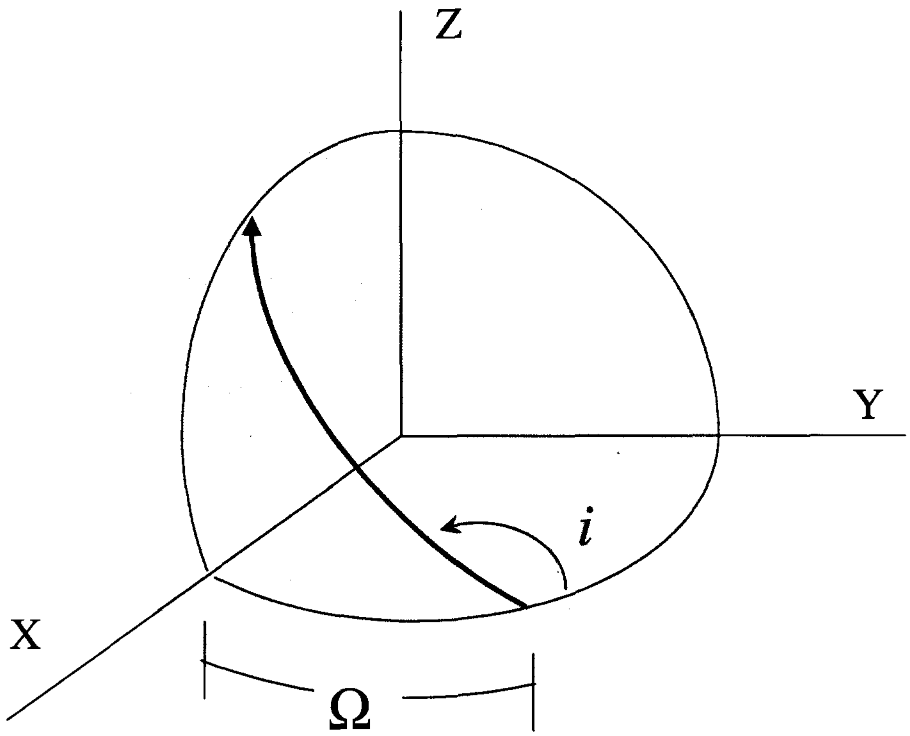
We used Figure 1 to define the Mean Local Time (MLT) for the ascending node. This is the angle, sometime measured in units of time, between the line running from the center of the earth to the sun and the position of the orbit's ascending node measured along the equator. In Figure 1, the MLT for the orbit shown is approximately 08:00 a.m. or with a right ascension of the ascending node, Ω, relative to the sun-line of -60 degrees. But Figure 1 is based on an idealized geometry where the earth was assumed to be in a circular orbit and the earth's polar axis was assumed to be perpendicular to the earth's orbit plane. We know that, in reality, the earth's orbit is not circular. Moreover, the earth's polar axis is inclined with respect to the ecliptic plane. Therefore, the idealized description of the geometry for a SS-O as given above is not correct and a more rigorous description is now required.

Just as is true for a lot of problems in celestial mechanics, the selection and use of a special coordinate system can provide keen physical insight into a problem, and in some cases facilitate a simple means to solution. For ease of describing the geometry associated with a SS-O, we now refer to Figure 5. In it is shown a coordinate system centered on the earth with the x-y plane coincident with the earth's equatorial plane and the z-axis coincident with the polar axis. We also represent in this figure an orbit plane for an arbitrary SS-O with its ascending node at an angle of Ω degrees reckoned in the x-y plane from the x-axis and with its inclination reckoned from the equatorial plane to the orbit plane (as shown). This, for all intents and purposes, is a relatively standard drawing of an earth-centered coordinate system with an arbitrary orbit plane represented.

Now we introduce a little complexity into the description. Let us further assume that the coordinate system rotates at a rate equal to the earth's Mean Motion. Moreover, the x-axis points to the "mean sun" as previously described. This yields three very important characteristics. First, the x-axis would remain pointed at the "mean sun" for all time, as a result of the rotation. Second, the position of the ascending node, i.e., the Mean Local Time, for the SS-O would remain fixed, again owing to the rotation. And third, the x-z plane would define the mean solar meridian on the sunlit side of the earth. The motion of the real sun in this coordinate system would be predominantly north-south due to changes in declination. As we will see, the sun's motion would also have an east-west component due to the Equation of Time.

With this coordinate system the problem of selecting the MLT for the orbit plane now becomes understood graphically. MLT is always in reference to the position of the mean sun, which (as in our original idealization) is always located along the +x-axis. So, a SS-O with a MLT equal to 08:00 a.m. still appears as represented in Figure 1. And a SS-O with a 12:00 noon MLT for its ascending node would have the line of nodes coincident with the x-axis. For a 06:00 p.m. MLT, the line of nodes would be coincident y-axis with the ascending node also situated on the +y-axis. This, therefore, provides the means for a mission analyst or system designer to readily visualize where the sun will be relative to the SS-O plane.

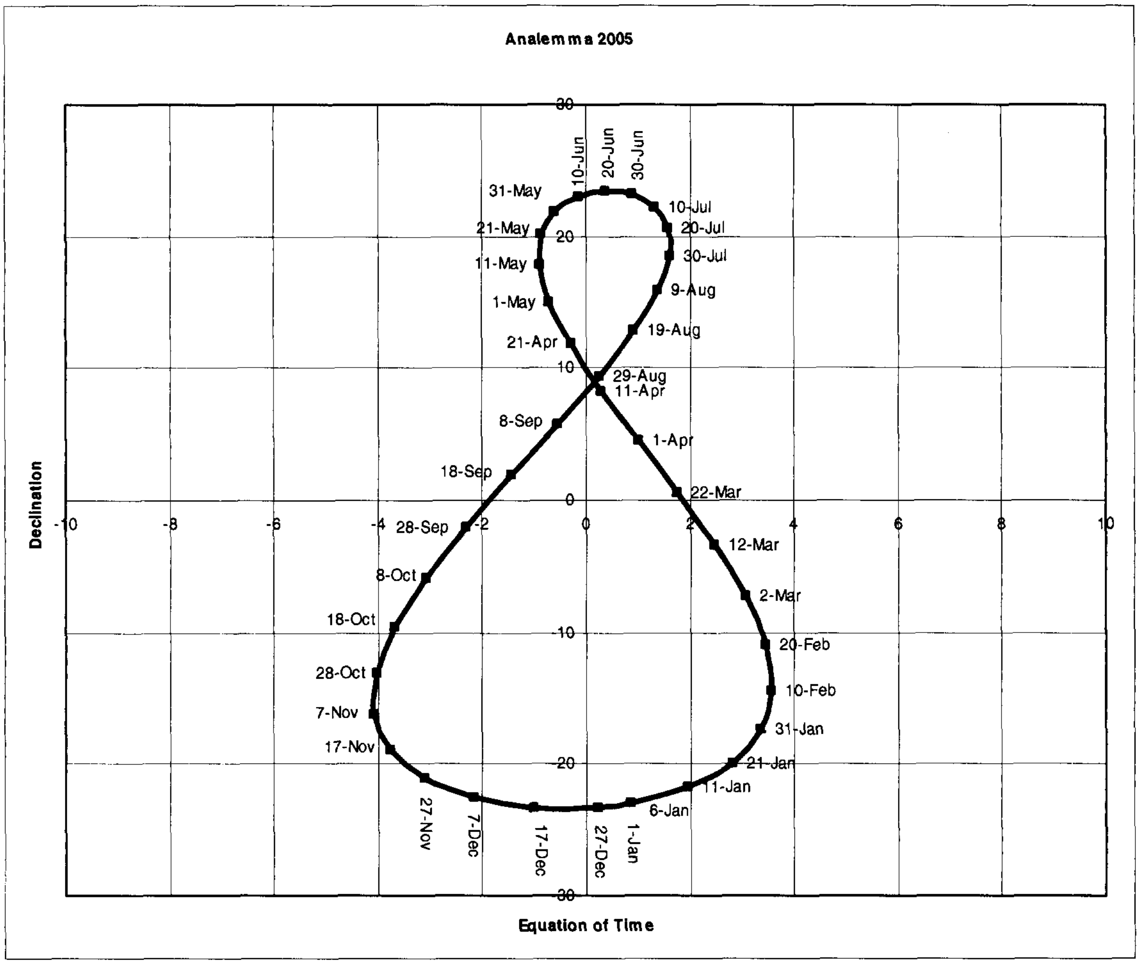
Figure 5: An Earth-Center Coordinate System with an Orbit Plane



However, we know from the previous discussion about **Reckoning Time** that the real sun does not confine its motion to be strictly north-south along a meridian. Rather, we have to take into account the Equation of Time, which describes the apparent sun's departures from the meridian containing the "mean sun". When this is done and if we simultaneously make a plot of the sun's changes in declination as a function of the Equation of Time, we get a figure known as the analemma, shown in Figure 6 for the year 2005. Now the analemma in declination – Equation of Time space appears as a distorted figure-eight and accurately shows the real sun's position with respect to the "mean sun" as a function of calendar day during the year 2005. Further remember that in declination – Equation of Time space the "mean sun" sits at the origin. The figure also shows how the true sun moves with respect to the "mean sun".

Interestingly, the shape and orientation of the analemma with respect to the equator and the mean solar meridian remain practically invariant from year to year. And one analemma would be good for all time were it not for leap years, which cause the sun's actual position on the analemma to hop slightly back-and-forth along the path from year to year.

Figure 6: Analemma in Declination - Equation of Time Space

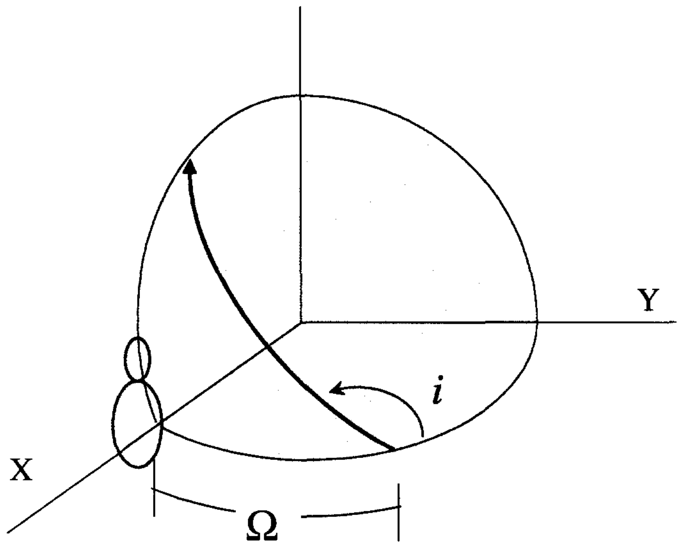


The analemma can be used to conveniently analyze the sun's motion relative to a SS-O's plane. This convenience is achieved by recognizing that we can move the figure of the analemma as shown in Figure 6 to a position around and centered on the x-coordinate axis of our system shown in Figure 5. When this is done, we get Figure 7.

So with the analemma, represented in the special coordinate system, enables us to visualize graphically how the sun moves with respect to the orbit plane, which remains fixed. For the sample orbit shown in the figure, it is clear that the real sun as it moves along the analemma path appears, at times, to approach and, at other times, to recede from the orbit plane as a function of calendar day in a year. Equally important, that same relative motion is repeated year after year.

Finally from Figure 7 it should be clear that, except for the 12:00 a.m. and 12:00 p.m. SS-Os, there is a dark side of the orbit, despite the sun's real movement along the analemma throughout the year. That is to say that the sun's position always remains on one side of the orbit plane or the other. The 12:00 a.m. and 12:00 p.m. SS-Os have the unique property of maximizing the time spent by a satellite in the earth's shadow per rev and can allow sun-light to fall on all sides of a satellite.

Figure 7: Analemma with respect to a SS-O



One can compute the unit vector that points from the earth to the sun, $\hat{s}$, for a given point on the analemma by reading off data from Figure 6. If δ is the sun's declination and ε is the value for the Equation of Time for a particular point on the figure-eight, the unit vector directed at the real sun in our special coordinate system is given by:

$$\hat{s} = (\cos\delta\cos\varepsilon)\hat{i} + (\cos\delta\sin\varepsilon)\hat{j} + (\sin\delta)\hat{k}$$

This equation will become handy in the next section.

IX. Calculating Beta-Angle for SS-Os

Using the special rotating coordinate system defined in the previous section, it is interesting to analyze properties of SS-Os related to how the direction of the real sun changes and even how the duration of time spent in the earth's shadow varies. We recognize that the parameter for a SS-O important to variations in the solar geometry is determined by the orbit's MLT and that angle is defined with respect to the mean solar meridian. On the other hand, the real sun moves along the analemma. This means that the real sun's position relative to the orbit plane for a SS-O varies over the course of a year causing the sun's direction to vary.

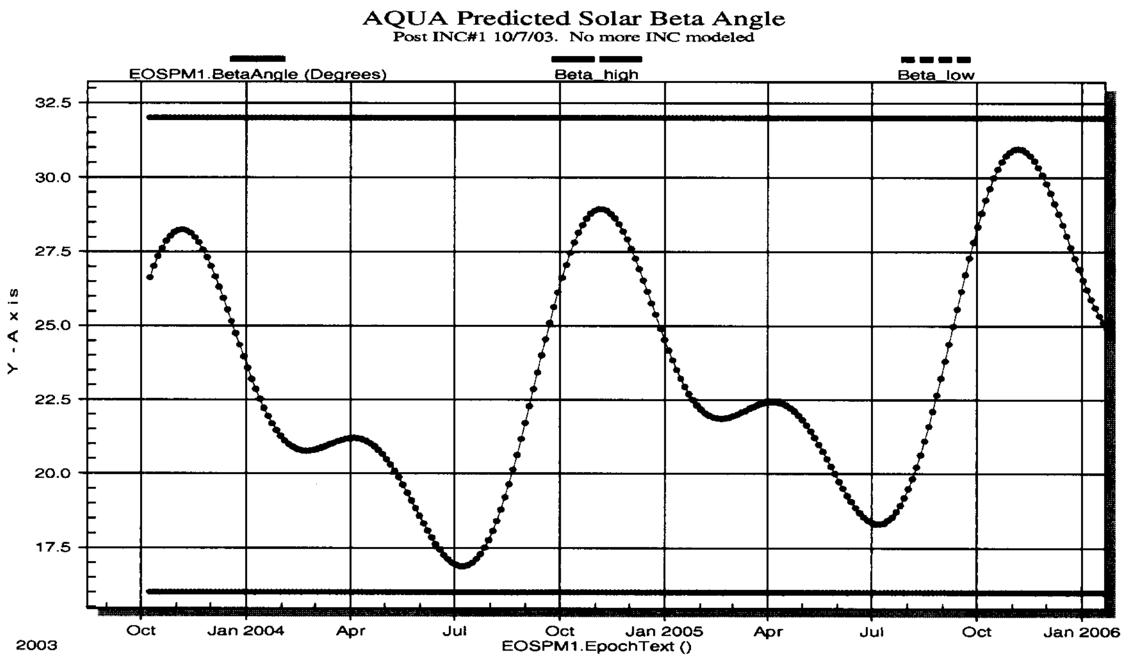
On the celestial sphere, the angular distance between the real sun's position on a given day and the closest point on the orbit plane is defined as the Beta-Angle. Said differently, the Beta-Angle is the angle between a vector directed at the sun and the perpendicular projection of that vector into the orbit plane. Beta-Angle can have either a positive or negative value, depending on which side of the orbit the sun is located. If the sun is on the same side of the orbit plane as the angular momentum vector, the Beta-Angle is positive; if on the opposite side, it is negative.

The usual way of computing Beta-Angle for an orbit as a function of time is to use a computer program, e.g., STK, SOAP, OASYS, Free-Flyer, (Refs. 11 – 14) which contains algorithms for both the solar ephemeris and for propagating the orbit's elements in time. For each day over a period of evaluation, the orbital elements are used to determine the angular momentum vector, $\hat{h}$, recognizing that the angular momentum vector changes direction in inertial space with time owing to orbital precession (and other perturbations). From the solar ephemeris, the vector directed to the sun, $\hat{s}$, is evaluated. Then by a simple dot-product of the two vectors (appropriately unitized), we readily obtain the Beta-Angle as:

$$\beta = \sin^{-1}\left(\hat{h} \bullet \hat{s}\right)$$

Frequently this calculation is repeated over an interval of interest to the system engineer to determine the maximum and minimum values of Beta-Angle and how it varies in between. This was done for the Aqua satellite from October 2003 through January 2006 with the results shown in Figure 8 (Ref. 15). (This figure represents an operational analysis of Aqua's orbit to predict how the Beta-Angle will evolve in time, with all order of effects and perturbations taken into account.) The first observation to be made about the figure is that the Beta-Angle for this sun-synchronous satellite has a sinusoidal pattern that repeats year to year. From the one can see that, even though the inclination and MLT remain essential constant over a one-year period, the value of the Beta-Angle ranges between 17 and 29 degrees in 2004 and between 18 and 31 degrees in 2005. Moreover, the cyclic pattern of 2004 is repeated in 2005 with two maxima and two minima in both years. And looking closely at the figure, it is apparent that from year to year the absolute minimum for Aqua's Beta-Angle occurs in the month of July. Likewise, the absolute maximum occurs in November.

Figure 8: AQUA Solar Beta-Angle Prediction



In order to understand the cause of this behavior, we must first describe Aqua's SS-O parameters. Aqua flies in a 16-day repeat with 233 revs, i.e., it's a 16D233R SS-O. From this designation and by use of equations previously provided, we quickly calculate the equatorial altitude as 705.3 km. And from Eq. 1 or Figure 2, we can quickly determine that the inclination is 98.2 degrees. Next we need a specification of Aqua's MLT.

The actual value of Aqua's MLT varies quadratic with time, owing to luni-solar perturbations to the orbital inclination. But for the system engineer who is more concerned about defining the orbit's sun-lighting conditions and sun-lighting effects on science, this quadratic variation is more of an orbit maintenance detail left to the astrodynamist after the basic value or range of values for MLT has been chosen. We can gain insight into how to go about selecting a desirable value for the MLT, which fixes the orientation of a SS-O, by studying the Aqua orbit. For Aqua, the MLT is allowed to vary in a range between 01:30 and 01:45 p.m. This corresponds to the ascending node being between 22.5 and 26.5 degrees with respect to the meridian of the mean sun, and this choice has provided the Aqua scientists with afternoon sun-lighting conditions that they desired. Thus, the Aqua orbit with respect to the rotating coordinates is similar to what is shown in Figure 7 for the arbitrary orbit. And from data associated with the generation of Figure 8 (Ref. 11), the average MLT for Aqua in the year 2004 was predicted to be 01:34:20 p.m. By 2005, the average MLT has increased to 01:40:30 p.m., or 25.13 degrees. We will use the 2005 value for our calculations here.

We now have values for inclination (98.2 degrees) and for the MLT (25.13 degrees) for the Aqua orbit (in 2005). We next want to see if we can use this information in computing the solar Beta-Angles and thereby develop an understanding of the shape and characteristic of the Beta-Angle curve shown in Figure 8. We shall do this by using the analemma for characterizing the solar movement relative to the orbit plane. First, we compute the orbit's angular momentum vector as:

$$\hat{h} = \left(\sin(i)\sin(\Omega)\right)\hat{i} - \left(\sin(i)\cos(\Omega)\right)\hat{j} + \cos(i)\hat{k}$$

where i is the inclination and Ω is the MLT angle. We assume that this vector remain fixed, i.e., constant, for the year 2005. We know from Figure 8 that the absolute minimum Beta-Angle occurs in July. So, we take from the analemma curve the values for declination and the Equation of Time of the sun on July 10th ($\delta = 22.00$ and $\varepsilon = 1.33$ degrees) and substitute them into the equation for the sun vector given in the previous section. When we take the arcsine of the dot-product of these two vectors, we obtain a value of approximately 18 degrees for β, in agreement with the value read off Figure 6. Thus, we have a point on the analemma corresponding to a point and date on the Beta-Angle curve in Figure 8. If we now refer to Figure 7, spotting our point on the analemma as before, and look at how the apparent angular distance to the orbit plane increases and shrinks as we move along the analemma, we will realize that from the July timeframe the sun's motion relative to the orbit plane slowly begins to increase and continues to increase until November 2005. In November it reaches a maximum angular separation from the orbit plane. Interestingly, this behavior is mapped out identically in Figure 8.

After November, again moving along the analemma, the angular separation begins to decrease through the month of February where it again reaches a local minimum. This behavior is also shown in Figure 8. After the February local minimum, the sun continues along the analemma increasing its separation from the orbit plane going through yet another local maximum during the April-May timeframe and then proceeds to decrease to the July minimum. From this, we see that the analemma allows us to completely characterize the time variations of the Beta-Angle for Aqua's SS-O.

Using the same analysis approach, the mission analyst can calculate the Beta-Angle profile for any arbitrary SS-O. First selecting the orbit altitude, which in turn specifies the inclination, and next selecting the MLT angle, it is possible to set up a graphic similar to Figure 7 that shows the analemma with respect to the SS-O plane. From this graphic and by examining the relative separations between the analemma and the orbit, the analyst can quickly visualize how the Beta-Angle will vary over a year. Therefore, without computer tools, it is easy to find the points of greatest and least angular separation between the analemma and the orbit, and then by picking selected points in between, compute a figure similar to Figure 8.

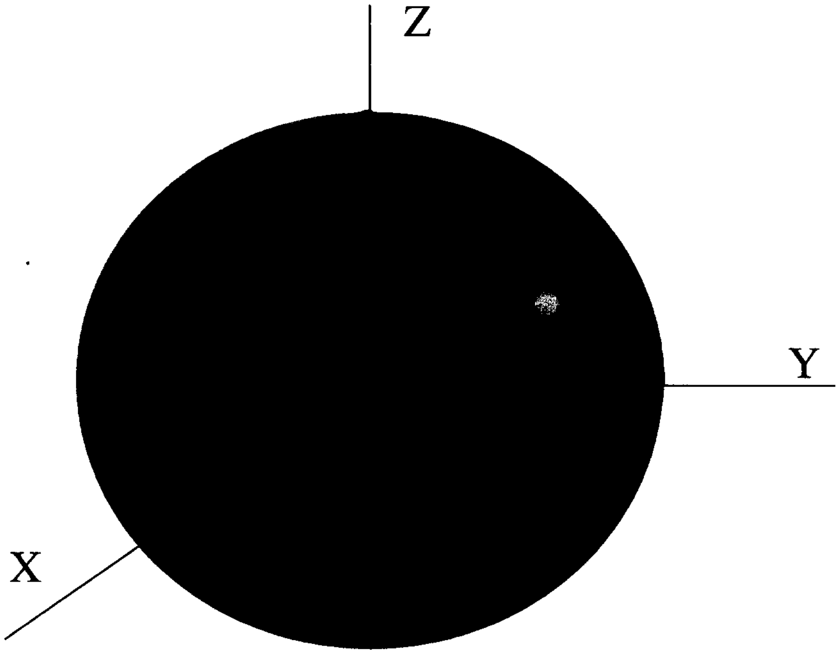
X. Calculating Shadow Time

As a final set of calculations often useful to the mission analyst or system engineer, we provide a quick algorithm for evaluating the time interval that a SS-O satellite spends in the earth's shadow given the Beta-Angle. Following the algorithm presented in Ref. 9, it is shown there that the time in shadow is a function of a satellite's orbital altitude and Beta-Angle at the time of evaluation. From the semi-major axis, a, we calculate one-half the angle subtended by the earth's figure as seen from the satellite, η, according to:

$$\eta = \sin^{-1}(a_e / a)$$

Referring to a coordinate system centered on the satellite with the z-axis coincident with the angular momentum and the x-y plane coincident with the orbit plane, we would see the figure of the earth centered at some point along the orbit plane. Let us take the position marked by the center of the earth as defining the direction of the x-axis relative to the coordinate system's origin. If we were standing on the satellite looking at the celestial sphere, we would see the edge of the figure of the earth circumscribing a small circle with the x-axis as its pole and with a radius equal to η. Anything that passes into that small circle would be occulted by the earth. Now if the Beta-Angle, β, is the angle between the sun and the orbit plane, the sun would be positioned at some arbitrary longitude measured along the orbit plane from the x-axis. β would be equivalent to the sun's latitude measured along a meridian line from the z-axis to the x-y plane. Moreover, because of the satellite's motion around the earth, the sun would appear to revolve around the z-axis as a pole circumscribing a small circle of radius equal to ($90° - \beta$). With this picture in mind, it is clear that if β is greater than η, then the figure of the sun would never be occulted by the figure of the earth as it revolved around. That is, the satellite would not enter into the earth's shadow. On the other hand, if β is less than η, then indeed the sun would be occulted by the figure of the earth. See Figure 9.

Figure 9: Calculating Shadow Time



Again following the algorithm outlined in Ref. 9, we note that, when β is less than η, the small circle that the sun moves along intersects the small circle defining the figure of the earth. When that occurs, we have the spherical triangle indicated in Figure 9. From the figure, it should be noted that the angle $\vartheta/2$ is just one-half of the mean anomaly angle that the sun must move through in order to transit behind the earth's figure. Solving this triangle for $\vartheta/2$, we get:

$$\cos(\vartheta/2) = \cos(\eta)/\cos(\beta)$$

Solving for ϑ, we get the time interval spent in the earth's shadow as:

$$\Delta t = \frac{\vartheta}{360^o} * P$$

Perhaps the situation of greatest interest is the calculation of the maximum shadow time. This clearly occurs for the smallest Beta-Angle. Performing a sample calculation for Aqua's orbit, we recall that the minimum Beta-Angle was ≈17 degrees. Solving the equation above for η, we get 64.2 degrees. Now, substituting these into the equation derived from the spherical triangle gives us:

$\vartheta/2 = 62.9°$ and $\Delta t = 125.6°/360° * P = 38.9$ minutes

Clearly, with the Beta-Angle profile computed as described in the previous section, specific values for β can then be used with this technique for obtaining a good estimate for the time spent by a sun-synchronous satellite in earth shadow. Moreover, by making the evaluation for both the max and min values for β it is possible to determine the max and min values of shadow time.

XI. Acknowledgements

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